# Simplicial Representation for Description Logics Knowledge Bases

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## Abstract

In order to solve semantical heterogeneity between dialogical agents, we propose to use adaptation algorithms that we developed in previous works. Since we model our dialogical agents as description logics KBases and that our adaptation algorithms are based on a topological knowledge representation, we explore the possible correspondences between description logics knowledge bases and this topological representation.

**Key Words** : Description Logics Knowledge Bases, Expressivity, Simplicial Complexes, Algebraic Topology, Adaptation.

## 1 Introduction

**Background : Semantical Heterogeneity** In the field of agents communication, we are interested in semantical heterogeneity arising between dialogical agents. We study this problem in the context of the InterViews project [11] which aims to put into dialogical interaction heterogeneous software applications embedded into HTML pages, via the Internet.

**Formalization in Description Logics** We modeled the dialogical agents as description logics KBases, and defined semantical heterogeneity between such agents[1]. Indeed, it appears that Description Logics (DL) are very well suited to model semantical heterogeneity problems [5], [10] : It is already used into the query management field [12]; and we know that they are a good way to express incomplete information, and thus useful when used for querying unknown databases [3].

**A Problem of Adaptation** In our study of semantical heterogeneity between description logics modeled agents it appears that the query management function must make use not only of matching tools for the explicit patterns contained into the queries, but that it also needs more elaborate algorithms to deal with heterogeneous *terms* of the query which are not explicit patterns. We propose to use the notion of *adaptation* from known terms of the local terminology of the agent interpreting the query towards the heterogeneous terms to interpret, before the classical application of matching tools.

This work has to be placed into the context of our preceding studies concerning analogy solving and structure adaptation [13], for which we already developed a model, called ESQIMO[2], based on a topological knowledge representation (CAT formalism [14]).

We claim that the use of the adaptation algorithms coming from our topological model is good and natural since we already showed [13] that analogy and especially adaptation had to be considered as a spatial transformation over a given spatial and structured representation.

[1] Article submitted to IBERAMIA 2000.

[2] See http:www.limsi.fr/Individu/erika/esqimo.html

**Aim of the Paper** For those computational purposes, we would like to show in this paper, that there is a correspondence between a description logics KBase and a topological formalization. The question is now to know if there is such a correspondence, and if it is well suited to express a DL KBase into the CAT formalism.

# 2 Preliminaries

## 2.1 Simplicial Representation

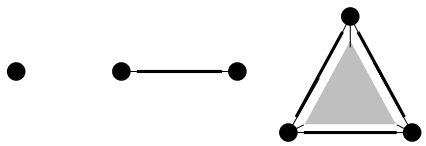
Topology studies objects and properties that are invariant under continuous deformations. Combinatorial topology focuses on the study of a finite sets of objects satisfying some spatial relations, and algebraic topology develops the application of algebraic tools to topological problems [8]. The Combinatorial Algebraic Topological (**CAT**) approach is thus attractive for constructivist models and applications.

### 2.1.1 Defining Simplicial Complexes

Simplicial Complexes (SC) are topological abstract structures that generalize the notion of graph. Indeed, all complexes of dimension less than 2 are graphs, and are thus interesting for computationnal purposes. The figure 1 shows the geometrical realization of simplexes of dimension resp. 0, 1 and 2 : they correspond to a vertice (point), an edge (segment) and a face (triangular). The face cannot be reduced to a portion of graph without loosing topological properties, while the simplexes of dimension 0 and 1 can.

The following definition is standard in algebraic topology [8]. An abstract simplicial complex is a couple $(V, K)$ where $V$ is a set of elements called vertices and $K$ is a set of finite parts of $V$ such that if $s \in K$, then all the parts $s' \subseteq s$ belongs also to $K$. The elements of $K$ are called abstract simplexes. The dimension of a simplex $s$ is equal to $Card(s)-1$. The dimension of the complex is the dimension of its biggest simplex.

Figure 1: Simplexes of dimension 0 (point), 1 (line) and 2 (triangular face).



(a) dim 0 (b) dim 1 (c) dim 2

A $p$-simplex $s$ is noted : $s = \langle v^0 v^1 ... v^p \rangle$, where $v^i \in V$. A complex is a set of sets closed for the inclusion and the intersection. Thus, simplicial complexes are particularly attractive to generalize semantic networks by keeping the possibility to express hierarchies like in a relational graph [14].

### 2.1.2 Representation of a Binary Relation

Atkin [2] proposed to represent a binary relation $\lambda$ between two sets with a simplicial complex: it is part of the the Q-Analysis. Q-Analysis have been used to model traffics, interactions between agents [4], etc.

Let $\Lambda$ be the incidence matrix of a binary relation $\lambda \subset A \times B$. Let $a \in A$, and the set $B_a$ of $b_i \in B$ such that $(a, b_i) \in \lambda$. The set $B_a$ can be directly read from $\Lambda$, as the $a$-column (see table 1).

We represent the elements $b_i$ of $B_a$ as vertices and $a$ as a simplex build on these vertices. The dimension of the simplex $S_a$ representing $a$ depends on the number of vertices in $B_a$.

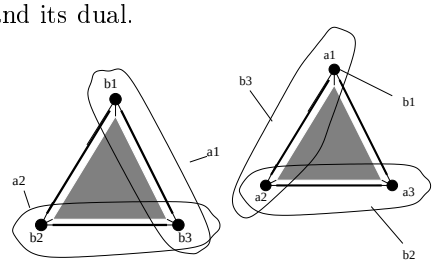
The whole matrix $\Lambda$ can then be represented as a simplicial complex containing all the simplexes representing each element $a_i \in A$, we note it $K_A(B, \lambda)$ (see figure 2 a).

Likewise, we can represent $\Lambda^{-1}$ with the dual simplicial complex $K_B(A, \lambda^{-1})$ (see figure 2 b). In this case, the elements $a_i$ are taken as vertices and the elements $b_i$ are represented as simplexes. We say that $K_A(B, \lambda)$ and $K_B(A, \lambda^{-1})$ are conjugates, they contain the

| $\lambda$ | $a_1$ | $a_2$ | $a_3$ |
|---|---|---|---|
| $b_1$ | 1 | 0 | 0 |
| $b_2$ | 0 | 1 | 1 |
| $b_3$ | 1 | 1 | 0 |

Table 1: Incidence matrix associated with $\lambda$.

Figure 2: Simplicial representation of $\lambda : A \rightarrow B$ and its dual.




same information but presented it in a dual way.

We say that two simplexes $\sigma_1$ and $\sigma_2$ are $q$-connected if there is a polygonal chain of dimension $q$ that connects $\sigma_1$ with $\sigma_2$.

Let $\alpha = (\sigma_0, \sigma_1, ..., \sigma_n)$ be a sequence of simplexes belonging to a complex $K$. It is called a polygonal chain of origin $\sigma_0$ and end $\sigma_n$ if for all couple $(\sigma_i, \sigma_{i+1})$, $\sigma_i \cap \sigma_{i+1} \neq \emptyset$. The dimension of $\alpha$ is the smallest dimension of $\sigma_i \cap \sigma_{i+1}$. Any $p$-simplex is $p$-connected to himself with a 0-chain.

Our model for adaptation in analogy solving problems [13], makes large use of this notion of polygonal chain : the transformation of a knowledge representation structure into another is modeled as a simplicial transformation along the simplexes of a polygonal chain.

### 2.1.3 Representation of a Set of Predicates

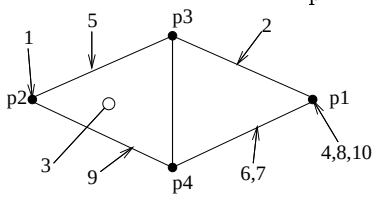
We extend this method to represent sets of predicates as simplicial complexes too. The idea, which is simple, is to take a set of predicates $P = \{p_1, p_2, ..., p_n\}$ and represent the binary relation $\lambda \subset A \times P$ such that $(a_i, p_j) \in \lambda$ if $p_j(a_i)$ holds.

Consider for example the set of integers $A = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and the

| | $p_1$ | $p_2$ | $p_3$ | $p_4$ |
|---|---|---|---|---|
| 1 | 0 | 1 | 0 | 0 |
| 2 | 1 | 0 | 1 | 0 |
| 3 | 0 | 1 | 1 | 1 |
| 4 | 1 | 0 | 0 | 0 |
| 5 | 0 | 1 | 1 | 0 |
| 6 | 1 | 0 | 0 | 1 |
| 7 | 0 | 1 | 1 | 0 |
| 8 | 1 | 0 | 0 | 0 |
| 9 | 0 | 1 | 0 | 1 |
| 10 | 1 | 0 | 0 | 0 |

Table 2: Incidence matrix associated with $\lambda$ in the first ten numbers example.

Figure 3: Simplicial Representation of a set of predicates for the numbers example.




set of predicates $P = \{p_1, p_2, p_3, p_4\} = \{\text{parity, oddity, primality, multiple of 3}\}$. The incidence matrix of $\lambda$ is then obviously the one given in table 2. We can represent the dual complex of $\lambda$, each element $a_i \in A$ being a simplex build with vertices $p_i \in P$. Figure 3 shows for example that the "observation" of the first ten numbers by this set of predicates results into a 1-simplex for the number 2, but results into the same 1-simplex for the elements $4, 8, 10$. this means that these numbers have exactly the same representation when we observe them *only* through these few predicates. A representation based upon simplicial complexes associates the same simplex to elements of $A$ that cannot be distinguished. In other words, two elements will be separated only if there is at least one predicate that allows the differentiation. The same situation occurs with the dual complex.

Two simplexes that have a smaller $k$-simplex in common are said to share a $k$-face. In terms of representation, this means that *they have $k$ features in common.* As Freska emphasized it, we call here for the use of discriminating

features rather than for precise characterization in terms of universally applicable reference system [6].

We can say that the identity of an element is represented by the features he shares with others and also by the ones that are specific to it [9].

## 2.2 Expressing First Order Logic formulae with Simplicial Complexes

To show the possible expression of description logics KBases into a simplicial representation, we first take a look at possible correspondences between first order logics (FOL) formulae and simplicial representations. Indeed, we know that all the knowledge one can express in a DL KBase can be expressed into FOL formulae.

This correspondence between first order logic formulae and simplicial complexes is crucial in order to give an intuition of how we could interpret each building bloc : constant, variable, predicate, and constructors, that are also important notions of DL.

### 2.2.1 Domain of Interpretation

Let $D$ be the domain of interpretation of all the first order logic (FOL) formulae, the semantics of a FOL formulae is then the validity domain of its expression. The domain is usually represented as points : we will represent them as vertices in the CAT formalism.

The constants and variables are usually interpreted as individuals of the world with or without identifiers attached to them, and represented as points. Likewise, in the CAT formalism, they are represented with 0-simplexes (vertices) of the abstract algebraic space.

### 2.2.2 Quantifications

The Universal Quantification $\forall x P(x)$, denotes the validity domain $D_P = D$ for the predicate $P$. The modelization of this formula in the CAT formalism corresponds to the maximal simplex including all the vertices of the domain $D$. We note this maximal simplex $S_D$.

The Existential Quantification $\exists x P(x)$ denotes a single individual, of undeterminate identifier. Its modelization in terms of CAT corresponds to a single undeterminate vertice.

### 2.2.3 First Order Predicates

**Using Simplexes** We can choose to represent the validity domain $D_P$ of a predicate $P$ by the simplex constructed on all the vertices belonging to the set $D_P$. In this case, the dimensionality of a simplex corresponds to the cardinality of the validity domain. But the problem is that we cannot represent n-ary predicates this way. The predicates represented with the CAT formalism in section 2.1.3 were all of arity 1, but this is not the case anymore in FOL.

**Using Complexes** Let $P$ be of arbitrary arity $n$ ; its validity domain is a set composed of $n$-uples. Its representation in terms of CAT is then a complex composed of the simplexes representing r each $n$-uple of $D_P$. For example, let $D = \{a, b, c, d\}$, and let the 2-ary predicate $R$ have the validity domain $D_R = \{(a, c), (a, b)\}$, its simplicial representation will then be $S_R = \langle\langle a, b\rangle, \langle a, c\rangle\rangle$, which are two connex edges.

### 2.2.4 Complementarity degree between two complexes

To represent composed formulae containing FOL constructors, we need to introduce the complementarity operator between simplicial domains.

Let the interpretation domain be $D = \{a, b, c, d\}$ and let the validity domain of the 1-ary predicate $P$, be $D_P = \{a, b, c\}$. Let us examine the complementary complex $D \setminus D_P$.

First, the maximal complex representing $D$ is given by : $S_D = \langle a, b, c, d\rangle$ with the standard notation (see section 2.1.1) . It means, that

the following faces are part of it : $S_D = \langle\langle a,b,c,d\rangle, \langle a,b,c\rangle, \langle b,c,d\rangle, \langle a,b,d\rangle, \langle a,c,d\rangle, \langle a,b\rangle, \langle a,c\rangle, \langle a,d\rangle, \langle b,c\rangle, \langle b,d\rangle, \langle c,d\rangle, \langle a\rangle, \langle b\rangle, \langle c\rangle, \langle d\rangle\rangle$.

First, we can take the complementary simplex by simply removing the face $\langle a,b,c\rangle$ corresponding to the simplex $S_P$ representing $D_P$ (since $P$ is here of arity 1). But, according to the definition of a complex, it has to contain all its faces, so we cannot keep the volume $\langle a,b,c,d\rangle$ if one of its face is missing. In consequence, we also have to remove it from $S_D$. Thus, the complementary is simply given by :

$$S^1_{\neg P} = \langle\langle b,c,d\rangle, \langle a,b,d\rangle, \langle a,c,d\rangle\rangle$$

Note that this expression is different from the simplex formed by all the vertices of $D$ not belonging to $D_P$. But if we consider the simplex formed by all the vertices not belonging to $D_P$, we get a different complementary simplex :

$$S^2_{\neg P} = \langle\langle a,d\rangle, \langle b,d\rangle, \langle c,d\rangle\rangle$$

We say that the two different complementary complexes resp. $S^1_{\neg P}$ and $S^2_{\neg P}$ are the complementary of $S_P$ into $S_D$ at dimension resp. 2 and 1. We can take the complementary of a $m$-simplex $S$ into a $n$-simplex $S'$ at any intermediate dimension between the dimension $n$ of $S'$ and the dimension $m$ of $S$.

### 2.2.5 Connectors

The four classical connectors of FOL are : negation $\neg$, conjunction $\wedge$, disjunction $\vee$ and implication $\rightarrow$. For the sake of simplicity, we only use predicates of arity 1 in our examples, thus their validity domains are only simplexes[3].

**Negation** $\neg$ Let us consider the FOL formula $\neg P(x)$. Its validity domain is given by the complementary $D_{\neg P} = D \setminus D_P$. We shall make use of the complementary at different degrees as mentioned above for expressing different significations in terms of knowledge representation. Thus simplicial complexes are more powerful.

**Conjunction** $\wedge$ Let us consider the FOL formula $F(x) = P(x) \wedge Q(x)$, whose validity domain is $D_F = D_P \cap D_Q$. The intersection between two simplexes is given by the *frontier*[4] between them : the shared faces.

**Disjunction** $\vee$ Let us consider the FOL formula $G(x) = P(x) \vee Q(x)$, whose validity domain is given by $D_G = D_P \cup D_Q$. The union between two simplexes may be the complex composed of the two simplexes or the "big" simplex constructed over the vertices belonging to at least of the simplexes. These two ways of constructing the union are close to the notion of degree of complementarity. They also have different meanings in terms of knowledge representation.

**Implication** $\rightarrow$ Let us consider the FOL formula $H(x) = P(x) \rightarrow Q(x)$ whose validity domain is given by $D_H = (D_P \cap D_Q) \cup D_{\neg P}$. The modelization of this domain in terms of CAT formalism is the union between the simplex $S_{\neg P}$ and the simplex . According to what we just said above, we have two ways of forming this unions. The simplex $S_{P \cap Q}$ is simply the frontier between $S_P$ and $S_Q$.

### 2.2.6 Discussion

It is possible to express every notion of FOL into the CAT formalism without any major difficulty. Each formula can be expressed with simplexes and complexes, and each constructor finds and expression in terms of topological operations on complexes. Moreover, we showed that we can make use of several different complementarities and unions between simplexes to represent different aspects of the same piece of knowledge, making SC more

[3] But we know how to deal with higher arities, as showed above.

[4] A frontier between two complexes is defined in algebraic topology as the set of all simplexes belonging to both complexes.

powerful from this knowledge representation point of view.

# 3 Simplicial Representation of a Description Logics KBase

Having established the correspondence between FOL constructors and SC, we can now give an expression for the basic components of a DL KBase [3]:

**Abox** : set of assertions about the individuals of the effective world, this world corresponds to the interpretation domain of the agent modeled by the particular KBase.

**Tbox** : set of descriptions introducing (only once) terms of the agents vocabulary.

These elements have to be kept separated since it is one of DL major powerful characteristics. Thus, we will propose a separate simplicial representation for ABoxes and for TBoxes.

## 3.1 Simplicial Representation of an ABox

Let $I$ be the set of effective individuals of the ABox, $C_e$ be the set of effective concepts of which the asserted individuals can be instances of, and $\lambda$ be the binary instantiation relation (member).

### 3.1.1 Example

Let us write the incidence matrix of $\lambda : I \rightarrow T$ . Each line is representable as a simplex according to the Q-Analysis method proposed by Atkin. Let us take the example from [10]:

$Human \leq ANYTHING$

$Man \leq Human$

$Woman \leq Human$

$leader \leq member$

$Team \doteq (\ Set \wedge (\forall\ member{:}\ Human)\ (\geq 2\ member))$

$SmallTeam \doteq (\ Team \wedge (\leq 5\ member))$

$ModernTeam \doteq (\ Team \wedge (\leq 4\ member)\ (\geq 1\ leader)\ (\forall\ leader{:}\ Woman))$

The following ABox makes use of this terminology[5] :

$(ModernTeam\ \#fabulous)$

$(Man\ \#joe)$

$(Man\ \#harry)$

$(Human\ \#mary)$

$(member\ \#tA\ \#joe)$

$(member\ \#tA\ \#harry)$

$(leader\ \#tA\ \#mary)$

The set of individuals is given by :

$I(A_0) = \{\#joe, \#harry, \#mary, \#fabulous\}$

The set of effective concepts is given by :

$C_e(A0, T_0) = \{Man, ModernTeam, Human\}$

### 3.1.2 Corresponding Simplicial Representation

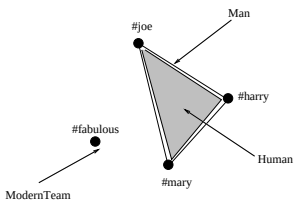
The instantiation relation $\lambda : I \rightarrow C_e$ is simply given by the following matrix :

| $\lambda$ | $ModernTeam$ | $Man$ | $Human$ |
|---|---|---|---|
| $\#joe$ | 0 | 1 | x |
| $\#harry$ | 0 | 1 | x |
| $\#mary$ | 0 | 0 | 1 |
| $\#fabulous$ | 1 | 0 | 0 |

Where the "x" represent $\lambda$-related elements that have been found to be related after a deductive process. Indeed, some individuals are instances of some effective concepts, but this fact is not explicit into the ABox, it has to be deduced by the system kernel. The simplicial representation of such a matrix is immediate, and given by figure 4.

[5] Where we note the individuals identifiers w

Figure 4: Simplicial Representation of the ABox.




#### 3.1.3 Analysis of the simplicial representation

The effective individuals are represented as 0-simplexes. The effective concepts are represented by simplexes of higher dimension but whose vertices are the individuals themselves. They represent the extension of the concept, the known occurrences of them. The dimensionality of a simplicial representation of an effective concept depends on the number of effective occurrences the KBase knows of this concept. The individuals being instances of several concepts belong to several simplexes and thus correspond to the frontier between the simplicial representations of those concepts. The inheritance relation is represented by the inclusion of simplexes (as in [14]).

### 3.2 Simplicial Representation of a TBox

#### 3.2.1 The Relations of a TBox

The relations that are used to introduce descriptions in a TBox, can be classified into three families :

1. Type specification : "is-a" . The simplicial representation of this kind of relation is done the same way as for the instantiation relation in the ABoxes (see section 3.1).

2. Concept specification : with the use of attributes. The simplicial representation corresponds to the representation of sets of predicates, and is trivial according to our extension of Q-Analysis given in section 2.1.3.

3. Structure specification : all the "part-of" relations. This kind of relationship plays a crucial role in the representation of the software components that we have to model as agents in InterViews : we study the simplicial representation of these structures in the following section.

#### 3.2.2 The part-of relation

Among the relations that we can represent with the spatial metaphor is the part-of relation. Many different part-of relations have been reported in [1], which classifies them in terms of homeomorphism, functionality and removability of the part.

In any case, all the part-of relations are binary relations and can be represented as simplexes with the Q-Analysis. Note that they correspond to the introduction of special constructors in the DL language (see [1][7]).

#### 3.2.3 Aggregates

In the field of the study of the part-of relations, aggregates should receive special attention [7]. Aggregates result from the explicit naming of a structure build from part-of relations over components. These aggregates present different granularities and may need the introduction of multi-dimensional representations to reflect their structure. The specific structure of an aggregate depends on the relationship between the parts and the whole but also on the relationship between the parts. CAT formalism gives us the opportunity to show these dependencies *all at once*.

We propose to represent the binary relation between the parts $P_i$ of the aggregate and the relations $r_i$ that relates them to the aggregate. If these parts are themselves aggregates, we introduce the relation $p_i \in \{r_i\}$ and consider the links between this part and the elementary parts. By doing this, we can represent composed aggregates (with different granularities). Let us take an example to clarify our proposition, here is an abstract TBox :

$a := x + y$

Figure 5: An abstract part-of relation + linking the three parts $h, i, j$ to the whole t $y$, and its simplicial representation.

| $y$ | $h$ | $i$ | $j$ |
|---|---|---|---|
| + | 1 | 1 | 1 |

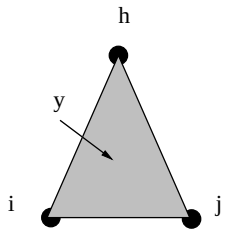
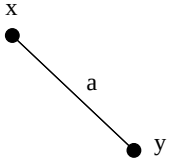



Figure 6: An abstract part-of relation + linking $x, y$ to $a$, and its simplicial representation.

| $a$ | $x$ | $y$ |
|---|---|---|
| + | 1 | 1 |




$y := h + i + j$

where the + operator symbolizes any of the part-of relations (we call it an abstract part-of relation). To represent this aggregate, we write the incidence matrix of the parts of $y$ and the relations that link them, see figure 5.

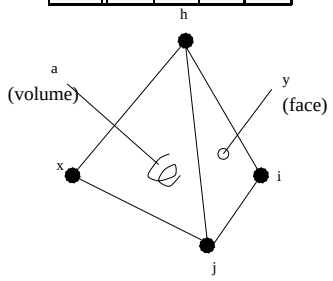
The simplicial representation of this relation is also given on figure 5. The aggregate $a$ is itself composed, and defined by the incidence matrix of figure 6, and represented as a 1-simplex.

Now, the question is how can we represent the compositionality of $y$ on the representation of $a$ ? It cannot be done *at the same level* by simply representing the four vertices $h, i, j$ and $x$ and then the 1-simplex $y$ over it as well as the 4-simplex representing $a$. Indeed, by doing this, we loose the structural information of $y$ being a sub-aggregate of $a$.

We propose then to introduce the "relation" $y$. And in general, each time that a part of $a$ is distinguished and named, we introduce its corresponding "relation", by which all the elementary parts of $a$ that are grouped into a sub-aggregate are linked together. In our example, we will say that "$h, i$ and $j$ are $y$-linked". This new way of considering $a$ is represented as shown on figure 7.

Figure 7: Proposition for a simplicial representation of a composed aggregate (different granularities).

| $a$ | $h$ | $i$ | $j$ | $x$ |
|---|---|---|---|---|
| + | 1 | 1 | 1 | 1 |
| $y$ | 1 | 1 | 1 | 0 |




This representation allows us to keep the structural information of the aggregate and also makes easier the direct reference to the named parts. By extension of this method, we could also refer to a sub-aggregate of any aggregate that hasn't been explicitly named. We could imagine that the part-of relations linking the sub-aggregates are not all of the same kind. In this case, the method remains the same, we only have to add labels to the edges of the simplexes (colored simplexes for example, see [14]).

# 4 Conclusion

First, we established a correspondence between first order logic formulae and simplicial complexes.

Second, we proposed to split the TBox relations into three families and give a procedure to build a simplicial representation for each family. Finally, we have four complementary and separated simplicial representations of a DL KBase : **Inheritance** ("is-a", relation between concepts of the TBox); **Struc-**

**ture** (relation between concepts and/roles of the TBox); **Features** (relations between predicates and concepts of the TBox); **Assertions** (relation between individuals and effective concepts of the ABox).

Note that the correspondence between DL KBases and simplicial representations lies on the fact that the denotational semantics is given in set theory, while simplicial complexes are also defined in those terms. By studying these correspondences, we showed that SC offer interesting properties in terms of powerfulness of the representation : for example, we introduced the notions of degree of complementarity, and the original representation of aggregates.

Finally, these results allow now us to directly apply our adaptation algorithm from the ESQIMO system on simplicial representation of the DL KBases modeling the agents in InterViews.

Our current work includes the implementation of algorithms to DL KBases and their testing on real semantically heterogeneous situations between software components.

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